BUSS 230 Managerial Economics

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Midterm Review Problems Solution

Problem 1

$$TR = 100Q - Q^2$$
$$TC = 2Q^3 - 5Q^2 + 36Q + 4$$

a) The profit function π(Q) is given by:

$$\pi(Q) = TR(Q) - TC(Q) = (100Q - Q^2) - (2Q^3 - 5Q^2 + 36Q + 4)$$
$$\pi(Q) = -2Q^3 + 4Q^2 + 64Q - 4$$

b) Profit is maximized where the first derivative of the profit function with respect to output equals zero, while the second derivative is negative, or:

$$\frac{d\pi}{dQ} = 0 \text{ and } \frac{d^2\pi}{dQ^2} < 0$$

$$\frac{d\pi}{dQ} = -6Q^2 + 8Q + 64 = 0$$

$$Q = \frac{-8 \pm \sqrt{8^2 - 4(-6)(64)}}{2(-6)} = 4$$

The other solution is negative. Verify that $\frac{d^2\pi}{dQ^2} < 0$ at Q = 4.

$$\frac{d^2\pi}{dQ^2} = -12Q + 8 = -12(4) + 8 = -40 < 0$$

Profit is therefore maximized at Q* = 4.

Problem 2

$$MC = 2Q^2 - 26Q + 80$$

Marginal cost is minimized where $\frac{dMC}{dQ} = 0$ and $\frac{d^2MC}{dQ^2} > 0$

$$\frac{dMC}{dQ} = 4Q - 26 = 0$$
$$Q = \frac{26}{4} = 4.5$$
$$\frac{d^2MC}{dQ^2} = 4 > 0$$

Therefore, marginal cost is minimal where Q* = 4.5.

Problem 3

$$\pi = 140X - 4X^2 - 4XY + 2Y^2 - 20Y$$
$$\frac{\partial\pi}{\partial X} = 140 - 8X - 4Y = 0$$
$$\frac{\partial\pi}{\partial Y} = -4X + 4Y - 20 = 0$$

Solving the above two equations simultaneously yields:
X* = 10 and Y* = 15.

Problem 4

$$\pi = 140X - 4X^2 - 4XY + 2Y^2 - 20Y$$
$$2X + Y = 40$$

Substitution method

$$Y = 40 - 2X$$

$$\pi = 140X - 4X^2 - 4XY + 2Y^2 - 20Y$$

$$\pi = 140X - 4X^2 - 4X(40 - 2X) + 2(40 - 2X)^2 - 20(40 - 2X)$$

$$\pi = 140X - 4X^2 - 160X + 8X^2 + 3200 + 8X^2 - 320X - 800 + 40X$$

$$\pi = 12X^2 - 300X + 3200$$

Maximizing profit means setting the first derivative of the profit function equal to zero.

$$\frac{d\pi}{dX} = 24X - 300 = 0$$

$$X^* = 12.5; Y^* = 15$$

Lagrangian method

$$L = 140X - 4X^2 - 4XY + 2Y^2 - 20Y + \lambda(2X + Y - 40)$$

$$\frac{\delta L}{\delta X} = 140 - 8X - 4Y + 2\lambda = 0$$

$$\frac{\delta L}{\delta Y} = -4X + 4Y - 20 + \lambda = 0$$

$$\frac{\delta L}{\delta \lambda} = 2X + Y - 40 = 0$$

First equation minus twice the second:

$$180 - 12Y = 0$$

So $Y^* = 15$

Plugging Y into third equation, $X^* = 12.5$

Plugging X and Y into second equation yields $\lambda^* = 10$

Problem 5

$$Q_X = 100 - 8P_X + 0.5P_Y + 2I$$

Using $P_X = 6$, $P_Y = 12$ and $I = 4.5$

$$Q_X = 100 - 8(6) + 0.5(12) + 2(4.5) = 67$$

a) $E_P = -8 \times \frac{6}{67} = -0.72.$

b) Demand is inelastic since $|E_P| < 1$.

c) At this point on the demand for product X (inelastic portion), the sign of the MR is negative.

d) Decreasing the price of X from 6 to 5 will cause TR to decrease because demand is inelastic.

e) At $P_X = 7$, $Q_X = 59$. The arc price elasticity of demand between $P_X = 6$ and $P_X = 7$ is equal to $\frac{59-67}{7-6} \times \frac{7+6}{59+67} = -0.83$

f) $E_{XY} = 0.5 \times \frac{12}{67} = 0.0896$. Y is a substitute product to X because a higher price for product Y yields a higher demand for X.

g) $E_I = 2 \times \frac{4.5}{67} = 0.13$. X is a normal good because a higher income level yields a higher demand for X.

Problem 6

$$P = 15 - 0.15Q$$

a) An increase in the price will lead to an increase in total revenues (as Terry would want) only when the prices lie on the inelastic portion of the demand.
There are two alternative approaches to answering this question.
The first consists of using marginal revenue to obtain the range of prices for which demand is elastic or inelastic. That is, first obtain the total revenue function as:

$$TR(Q) = P \times Q = (15 - 0.15Q) \times Q = 15Q - 0.15Q^2$$

This implies that the marginal revenue function is:

$$MR(Q) = \frac{dTR}{dQ} = 15 - 0.3Q$$

We know that demand is elastic when MR > 0, inelastic when MR < 0 and unitary elastic (midpoint of the demand curve) where MR = 0.

$$MR(Q) = 0 \rightarrow 15 - 0.3Q = 0 \text{ or } Q = 50$$

The corresponding price is $P = 15 - 0.15(50) = \$7.5$. That is, at Q = 50 and P = $7.5, demand is such that $|E_P| = 1$. This implies that for Q < 50, or P > $7.5, demand is elastic and $|E_P| > 1$. This means that a price of $9 is on the elastic portion of demand. If Terry raises her price above $9, this will lead to decrease in total revenues. Therefore she should not increase the price.

Alternatively, one can directly compute the point price elasticity of demand at P = $9.
At P = $9, the point price elasticity of demand is given by:

$$E_P = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$$

$$P = 15 - 0.15Q \; therefore \; Q = 100 - 6.667P$$

$$\frac{\Delta Q}{\Delta P} = -6.667$$

At P = $9, Q = 40. $E_P = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q} = -6.667 \times \frac{9}{40} = -1.5$. Given that $|E_P| > 1$ (a price of $9 is on the elastic portion of demand), a price increase will lead to a decrease in total revenues since the new price will also lie over the elastic portion of demand.

b) Only the first approach will be used here. Given P = 22- 0.22Q, the marginal revenue function is MR(Q) = 22-0.44Q. MR = 0 for Q = 50 and P = $11 and this corresponds to the unitary elastic point (midpoint) $|E_P| = 1$ of the demand. If Terry raises her price anywhere from $9 to $11, she would be operating over the inelastic portion of the demand and total revenues will increase. Terry should therefore increase her price for haircuts.

Problem 7

$$Q = 20 - 0.5P$$

a) The inverse demand function is given by expressing P as a function of Q

$$-0.5P = Q - 20 \text{ or}$$

$$P = \frac{(20 - Q)}{0.5} = 40 - 2Q$$

b) The total revenue function is given by:

$$TR = P \times Q = (40 - 2Q) \times Q = 40Q - 2Q^2$$

c) The marginal revenue function is given by:

$$MR = \frac{dTR}{dQ} = 40 - 4Q$$

d) The quantity, Q*, at which total revenue is maximized is obtained by setting MR equal to zero.

$$MR = 40 - 4Q = 0$$

Therefore, Q* = 10.

e) No, total revenue maximization does not qualify as an example of constrained optimization, since there is no constraint to account for (a cost constraint for example).

Problem 8

$$P = 3,000 - 40Q$$

a) The total revenue function is given by:

$$TR = P \times Q = (3,000 - 40Q) \times Q = 3,000Q - 40Q^2$$

The marginal revenue function is equal to the first derivative of the total revenue function with respect to output, or:

$$MR = \frac{dTR}{dQ} = 3,000 - 80Q$$

b) The demand for the firm's product is price elastic when MR > 0, i.e. when:

$$MR = 3,000 - 80Q > 0 \; or \; Q < \frac{3,000}{80} = 37.5$$

Plugging Q < 37.5 back into the demand function yields:

$$P > \$1,500$$

c) Total revenue is maximized when marginal revenue equals zero, i.e., when Q = 37.5 and P = $1,500.

Problem 9

a) These goods are extremely close substitutes. A 1% increase (decrease) in the price of one good results in a 1.2% increase (decrease) in the quantity demanded for the other.

b) $1.2 = \frac{\%\Delta Q}{5\%}$. Therefore, %ΔQ = 6%. This implies that the demand for the good rises by 6% as a result of a 5% increase in the price of the substitute.

Problem 10

$$Q_d = 5{,}000 - 50P$$

a) By plugging the value of P into the demand function we get:
for P = $10, Q = 5,000 – 50(10) = 4,500
for P = $20, Q = 5,000 – 50(20) = 4,000
for P = $30, Q = 5,000 – 50(30) = 3,500

b) The arc price elasticity is given by:

$$E_P = \frac{Q_2 - Q_1}{P_2 - P_1} \times \frac{P_2 + P_1}{Q_2 + Q_1}$$

The arc price elasticity between $10 and $20 is equal to:

$$E_P = \frac{4{,}000 - 4{,}500}{20 - 10} \times \frac{20 + 10}{4{,}000 + 4{,}500} = -0.176$$

The arc price elasticity between $20 and $30 is equal to:

$$E_P = \frac{3{,}500 - 4{,}000}{30 - 20} \times \frac{30 + 20}{3{,}500 + 4{,}000} = -0.333$$

c) The point price elasticity is given by:

$$E_P = slope\ of\ demand\ curve \times \frac{P}{Q}$$

At P = $10: $E_P = -50 \times \frac{10}{4{,}500} = -0.111$

At P = $20: $E_P = -50 \times \frac{20}{4{,}000} = -0.250$

At P = $10: $E_P = -50 \times \frac{30}{3{,}500} = -0.429$

Problem 11

The arc price elasticity can be computed using:

$$E_d = \frac{\Delta Q}{\Delta P} \times \frac{(P_1 + P_2)}{(Q_1 + Q_2)}$$

The total and marginal revenues are equal to:

$$TR = P \times Q \text{ and}$$

$$MR = \frac{\Delta TR}{\Delta Q}$$

Note that the changes considered here are changes of 10 units, so that you have to divide changes in TR by 10 to get the MR.

Price P ($)	**Quantity Q_d (pounds of steak)**	**Arc Elasticity E_d**	**Total Revenue ($)**	**Marginal Revenue ($/unit)**
$12	30	NA	360	NA
$11	40	-3.290	440	8
$10	50	-2.330	500	6
$9	60	-1.727	540	4
$8	70	-1.308	560	2
$7	80	-1.000	560	0
$6	90	-0.765	540	-2
$5	100	-0.579	500	-4
$4	110	-0.429	440	-6

Problem 12

$$Q = 250{,}000 - 500P - 1.50M - 240P_R$$

a) Plugging the values P = \$200, M = \$60,000 and P_R = \$100 yields:

$$Q = 250{,}000 - 500(200) - 1.50(60{,}000) - 240(100) = 36{,}000$$

b) The price elasticity of demand is given by:

$$E_P = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q} = -500 \times \frac{200}{36{,}000} = -2.77$$

The above equation uses the fact that ΔQ/ΔP, the slope of the demand function with respect to P, is equal to -500.

c) The income elasticity of demand is given by:

$$E_M = \frac{\Delta Q}{\Delta M} \times \frac{M}{Q} = -1.5 \times \frac{60{,}000}{36{,}000} = -2.5$$

Again, the above equation uses the fact that ΔQ/ΔM, the slope of the demand function with respect to M, is equal to -1.5.
Good X is inferior since an increase in M, leads to a decrease in Q.
The effect of a 4% increase in income can be calculated using:
E_M = %ΔQ/%ΔM = -2.5. Therefore, %ΔQ/4% = -2.5, or %ΔQ = -10%. This implies that a 4% increase in come leads to a 10% decrease in demand.

d) The cross-price elasticity is given by:

$$E_{XR} = \frac{\Delta Q}{\Delta P_R} \times \frac{P_R}{Q} = -240 \times \frac{100}{36{,}000} = -0.6667$$

The above computation uses the fact that ΔQ/ΔPR, the slope of the demand function with respect to R, is equal to -240.
The negative sign of the cross-price elasticity between X and R indicates that they are complementary goods, since an increase in P_R leads to a decrease in the demand for X.
To assess the effect of a 5% decrease in P_R on the demand for X we use:
E_{XR} = %ΔQ/%ΔP_R = -0.6667 so that %ΔQ/-5% = -0.6667 implying %ΔQ = 3.335%. This means that a 5% decrease in the price of R leads to 3.335% increase in the demand for X.

Problem 13

The estimated market demand for good X is:

$$\hat{Q} = 70 - 3.5P - 0.6M + 4P_Z$$

Where $\hat{Q}$ is the estimated number of units of good X demanded, P is the price of the good, M is income, and P_Z is the price of the related good Z. (all parameter estimates are statistically significant at the 1% level.)

a) X is an inferior good. The negative sign of $\Delta Q/\Delta M = -0.6$ implies that an increase in income, M, leads to decrease in demand Q.

b) X and Z are substitutes. The positive sign of $\Delta Q/\Delta P_Z = 4$ indicates that the goods are substitutes. An increase in P_Z leads to an increase in the demand for X.

c) Plugging P = 10, M = 30, and $P_Z = 6$ into the demand function we get:

$$Q = 70 - 3.5(10) - 0.6\,(30) + 4(6) = 41$$

Therefore,

$$E_P = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q} = -3.5 \times \frac{10}{41} = -0.85$$

$$E_{XZ} = \frac{\Delta Q}{\Delta P_Z} \times \frac{P_Z}{Q} = 4 \times \frac{6}{41} = 0.5854$$

$$E_M = \frac{\Delta Q}{\Delta M} \times \frac{M}{Q} = -0.6 \times \frac{30}{41} = -0.439$$

Problem 14

$$Q = a + bP + cM + dP_R$$

a) Law of demand stipulates that b should be negative (there's a negative relationship between the price of the good and quantity demanded). $\hat{b}$ is indeed negative as is predicted theoretically and equals -6.50.

b) $\hat{c}$ = ΔQ/ΔM=0.13926 > 0 which implies that the good is normal since demand increases as income increases.

c) The goods are complementary since $\hat{d}$ = -10.77= ΔQ/ΔPR < 0. This implies that an increase in the price of R leads to a decrease in the demand for X.

d) First, using p-values:
(On an exam, you should state the null and alternative hypotheses and provide details similar to the ones provided here)
Decision rule: Reject H_0 if p-value < α
α= 5% = 0.05
First, starting with the intercept, the null and alternative hypotheses are:
H_0: a = 0
H_1: a ≠ 0
p-value for , $\hat{a}$ is 0.0001 < 0.05
This implies that we reject H_0, $\hat{a}$ is significant.

To test the significance of own price of the good, we state the null and alternative hypotheses:
H_0: b = 0
H_1: b ≠ 0
p-value is 0.0492 < 0.05. This implies we reject H_0, so that the price of the good significantly affects quantity demanded.

To test the significance of income, we state the null and alternative hypotheses:

H_0: c = 0

H_1: c ≠ 0

p-value is 0.0001 < 0.05. This implies we reject H_0, so that income significantly affects demand.

Regarding the price of the related good R, the null and alternative hypotheses are:

H_0: d = 0

H_1: d ≠ 0

p-value is 0.0002 < 0.05. This implies we reject H_0, so that the price of related good R significantly affects demand.

Now, we test the same hypotheses stated above, using t-statistics:
Decision rule: Reject H_0 if the calculated t-statistic, in absolute value, exceed the critical value.

The calculated t-statistics are:

$$t_{\hat{a}} = \frac{\hat{a}}{se(\hat{a})} = \frac{68.38}{12.65} = 5.41$$

$$t_{\hat{b}} = \frac{\hat{b}}{se(\hat{b})} = \frac{-6.50}{3.15} = -2.06$$

$$t_{\hat{c}} = \frac{\hat{c}}{se(\hat{c})} = \frac{0.1392}{0.0131} = 10.63$$

$$t_{\hat{d}} = \frac{\hat{d}}{se(\hat{d})} = \frac{-10.77}{2.45} = -4.40$$

Critical value at α = 5% with degrees of freedom df = (n − k) = 24 – 4 = 20 is obtained from the t-table as: $c_{\alpha/2,n\text{-}k} = 2.086$

(Note n = 24 as shown next to observations on the computer output whereas k = 4).

$|t_{\hat{a}}|$ = 5.41 > 2.08 so that we reject H_0, meaning that the intercept is significant.

$|t_{\hat{b}}|$ =2.06 < 2.08 meaning we do not reject H_0. This implies that the price of the good does not affect the quantity demanded significantly.

$|t_{\hat{c}}|$ =10.63 > 2.08. We reject H_0, meaning that income affect demand significantly.

$|t_{\hat{d}}|$ =4.40 > 2.08. We reject H_0, meaning that the price of good R affects demand significantly.

Note that using p-values and t-tests leads to different conclusions regarding the significance of the price of the good. This is an inconsistent result and should not typically occur.

e) The null and alternative hypotheses are:
H_0: a = b = c = d =0
H_1: H_0 is not true

The regression output shows that R^2 = 0.8118. We also have k = 4. To test the null of joint insignificance, we need to compute the F-statistic as:

$$F = \frac{R^2/(k-1)}{(1-R^2)/(n-k)} = \frac{0.8118/(4-1)}{(1-0.8118)/(24-4)} = 28.7872$$

The appropriate critical value is obtained from the F-table at a significance level α = 5% as F_c = 3.10 (3 degrees of freedom for the numerator and 20 degrees of freedom for the denominator).

The decision rule is to reject H_0 if $F > F_c$. Since 28.7872 > 3.10, the null hypothesis is rejected. This implies that the variables included in this regression are jointly significant.

The decision rule using p-values is to reject the null if p-value < α. Given that the p-value associated with the F-statistic is 0.0001 < 0.05, H_0 is rejected using p-values and the variables are jointly significant.

f) By plugging P = 225, M = 24,000 and P_R = 60 into the demand function we get the quantity:

$$Q = 68.38 - 6.50 \times 225 + 0.13926 \times 24{,}000 - 10.77 \times 60 = 1{,}301.92$$

1) $E_P = \frac{\Delta Q}{\Delta P} \times \frac{\mathrm{P}}{Q} = \hat{b} \times \frac{\mathrm{P}}{Q} = -6.50 \times \frac{225}{1301.92} = -1.12$

2) $E_M = \frac{\Delta Q}{\Delta M} \times \frac{\mathrm{M}}{Q} = \hat{c} \times \frac{\mathrm{M}}{Q} = 0.1392 \times \frac{24{,}000}{1301.92} = 2.57$

3) $E_{XR} = \frac{\Delta Q}{\Delta P_R} \times \frac{\mathrm{P}_R}{Q} = \hat{d} \times \frac{\mathrm{P}_R}{Q} = -10.77 \times \frac{60}{1301.92} = -0.49$

Problem 15

$$Q = a + bP + cM + dP_R$$

a) First, using p-values:
Decision rule: Reject H_0 if p-value < α
α = 5% = 0.05 (since it has not been specified)
First, starting with the intercept, the null and alternative hypotheses are:
H_0: a = 0
H_1: a ≠ 0
p-value for $\hat{a}$ is 0.0716 > 0.05. This implies that we do not reject H_0, or that the intercept is not significant.

To test the significance of own price of the good, we state the null and alternative hypotheses:
H_0: b = 0
H_1: b ≠ 0
p-value for $\hat{b}$ is 0.0093 < 0.05. This implies we reject H_0, so that the price of the good significantly affects quantity demanded.

To test the significance of income, we state the null and alternative hypotheses:
H_0: c = 0
H_1: c ≠ 0
p-value for $\hat{c}$ is 0.0009 < 0.05. This implies we reject H_0, so that income significantly affects demand.

Regarding the price of tennis rackets, the null and alternative hypotheses are:
H_0: d = 0
H_1: d ≠ 0
p-value for $\hat{d}$ is 0.006 < 0.05. This implies we reject H_0, so that the price of tennis rackets significantly affects demand.

Note that the same conclusions would hold if we use a level of significance of 1%. At the 10% level, all the variables including the intercept would be significant.

The signs of the parameter estimates are consistent with demand theory: $\hat{b}$ is negative as the law of demand predicts, $\hat{c}$ is positive indicating tennis balls are a normal good and we expect for most goods, and $\hat{d}$ is negative as it should be for complements (tennis balls and tennis rackets).

Now, we test the same hypotheses stated above, using t-statistics: Decision rule: Reject H_0 if the calculated t-statistics, in absolute value, exceed the critical value.

The calculated t-statistics are:

$$t_{\hat{a}} = \frac{\hat{a}}{se(\hat{a})} = \frac{425{,}120.0}{220{,}300.0} = 1.92$$
$$t_{\hat{b}} = \frac{\hat{b}}{se(\hat{b})} = \frac{-37{,}260.6}{12{,}587} = -2.96$$
$$t_{\hat{c}} = \frac{\hat{c}}{se(\hat{c})} = \frac{1.49}{0.3651} = 4.08$$
$$t_{\hat{d}} = \frac{\hat{d}}{se(\hat{d})} = \frac{-1{,}456.0}{460.75} = -3.16$$

Critical value at $\alpha = 5\%$ with degrees of freedom df = (n – k) = 20 – 4 = 16 is obtained from the t-table as: $c_{\alpha/2,n-k} = 2.12$ (Note n = 20 as shown next to observations on the computer output whereas k = 4).

$|t_{\hat{a}}| = 1.92 < 2.12$ so we do not reject H_0, meaning that the intercept is not significant.

$|t_{\hat{b}}| = 2.96 > 2.12$ so we reject H_0, meaning that the price of the good affects quantity demanded significantly.

$|t_{\hat{c}}|$ =4.0811 > 2.12 so we reject H_0, meaning that income affect demand significantly.

$|t_{\hat{d}}|$ =3.16 > 2.12 so we reject H_0, meaning that the price of tennis rackets affects demand significantly.

The decisions regarding the significance of the variables that we get from p-values and t-statistics are consistent.

b) Denote by $c_{\alpha/2,n-k}$ the critical value corresponding to a 99% confidence interval (that is, at an α = 1%). Then, the 99% confidence intervals for b, c and d are given respectively by:

$$\left[\hat{b} - se(\hat{b}) \times c_{\frac{\alpha}{2},n-k}; \hat{b} + se(\hat{b}) \times c_{\frac{\alpha}{2},n-k}\right]$$
$$\left[\hat{c} - se(\hat{c}) \times c_{\frac{\alpha}{2},n-k}; \hat{c} + se(\widehat{bc}) \times c_{\frac{\alpha}{2},n-k}\right]$$
$$\left[\hat{d} - se(\hat{d}) \times c_{\frac{\alpha}{2},n-k}; \hat{d} + se(\hat{d}) \times c_{\frac{\alpha}{2},n-k}\right]$$

Here α = 1% and n - k = 16 so that $c_{\alpha/2,n-k}$ = 2.921.

Therefore, for the population parameter b, we have:
$[-37{,}260.6 - 12{,}587 \times 2.921;\ -37{,}260.6 + 12{,}587 \times 2.921]$ or
$[-74{,}027.2270;\ -493.9730]$

Similarly, for the population parameter c, we have:
$[1.49 - 0.3651 \times 2.921; 1.49 + 0.3651 \times 2.921]$ or
$[0.4235; 2.5565]$

Similarly, for the population parameter d, we have:
$[-1{,}456.0 - 460.75 \times 2.921; -1{,}456.0 + 460.75 \times 2.921]$ or
$[-2{,}801.8508; -110.1493]$

The decision rule using confidence intervals is to reject H_0 if the hypothesized value lies outside the interval. In our case, the hypothesized value is zero. Since zero lies outside the

99% confidence interval for b, H_0 is rejected. Similarly, the null hypothesis H_0 is rejected using the other two 99% confidence intervals for c and d since the hypothesized value lies outside the 99% confidence interval.

c) The null and alternative hypotheses are:
H_0: a = b = c = d =0
H_1: H_0 is not true

The regression output shows that $R^2 = 0.8435$. We also have k = 4. To test the null of joint insignificance, we need to compute the F-statistic as:

$$F = \frac{R^2/(k-1)}{(1-R^2)/(n-k)} = \frac{0.8435/(4-1)}{(1-0.8435)/(20-4)} = 28.6939$$

The appropriate critical value is obtained from the F-table at a significance level α = 5% as $F_c = 3.24$ (3 degrees of freedom for the numerator and 16 degrees of freedom for the denominator). The decision rule is to reject H_0 if $F > F_c$. Since 28.6939 > 3.24, the null hypothesis is rejected. This implies that the variables included in this regression are jointly significant.

d) As indicated by R^2, 84.35% of the variation of cans of tennis balls sold quarterly is "explained" by the independent variables in this regression. The unexplained variation is therefore 15.65%.

e) The adjusted R^2 of this regression can be computed as:

$$\overline{R^2} = 1 - (1 - R^2)\left(\frac{n-1}{n-k}\right) = 1 - (1 - 0.8435)\left(\frac{20-1}{20-4}\right) = 0.8142$$

f) Plugging those values into the demand equation we get:
$425{,}120 + (-37{,}260.6 \times 1.65) + (1.49 \times 24{,}600) + (-1{,}456 \times 110) =$ 240,134 cans per quarter.

g) The elasticities can be computed as:

$$E_P = -37{,}260.6 \times \frac{1.65}{240{,}143} = -0.256$$

$$E_M = 1.49 \times \frac{24{,}600}{240{,}143} = 0.153$$

$$E_{XR} = -1{,}456 \times \frac{110}{240{,}143} = -0.667$$

h) % ΔQ /-15% = -0.256; implying that sales rise by 3.84% when the price of tennis balls decreases by 15%.

i) % ΔQ / 20% = -0.153; implying that sales rise by 2.98% due to an increase in average household income by 10%.

j) % ΔQ / 25% = -0.667; implying that sales fall by 16.67% due to an increase in the average price of tennis rackets by 25%.

Problem 16

$$\hat{Q} = 8000 - 25P - 0.12M - 30P_G$$

a) Yes, the sign of the estimated coefficient is consistent with theory. The law of demand holds that price and quantity demanded are inversely related and this is what we observe in this estimated demand function.

b) Given that the estimated coefficient on income is -0.12, X is an inferior good. An increase in income leads to a decrease in the demand for X.

c) Goods X and G are complements. This is indicated by the sign of the coefficient associated with P_G (the coefficient is -30). This, in turn, indicates that an increase in the price of G leads to a decrease in the demand for X.

d) The predicted quantity of good X is equal to:

$$\hat{Q} = 8000 - 25 \times 12 - 0.12 \times 30{,}000 - 30 \times 50 = 2{,}600$$

e)

1) Own price elasticity $E_P = -25 \times \frac{12}{2{,}600} = -0.1154$
2) Cross-price elasticity $E_{XG} = -30 \times \frac{50}{2{,}600} = -0.5769$
3) Income elasticity $E_M = -0.12 \times \frac{30{,}000}{2{,}600} = -1.3846$

f) 65% of the variation in quantity demanded is "explained" by variation in the independent variables included in this regression. The remaining 35% of the variation remains unexplained.

g) Given that n = 25 and k = 4, and $R^2 = 0.65$:

$$F = \frac{R^2/(k-1)}{(1-R^2)/(n-k)} = \frac{0.65/(4-1)}{(1-0.65)/(25-4)} = 12.976$$

h) The appropriate critical value at the 5% level is $F_c = 3.07$.
The null hypothesis is: H_0: a = b = c = d =0
The alternative hypothesis is: H_1: H_0 is not true
We reject H_0 if F-statistic > critical value. In this case 12.976 > 3.07 so the null hypothesis is rejected at the 5%, and the coefficients are jointly significant.

i) A 20% increase in income will lead to 20%×1.38= 27.6% decrease in the demand for X.

j) A 15% increase in the price of G leads to 15%×0.58 = 8.7% decrease in the demand for X.

Problem 17

$$Q_t = a + bt + c_1D_1 + c_2D_2 + c_3D_3$$

a) In order to check whether shoe sales exhibit an upward trend we need to consider the sign and the significance of the coefficient associated with the time trend, namely b. The estimated coefficient $\hat{b}$ is positive, meaning there is an upward trend in sales. Next, we need to check the significance of the population coefficient.

In other words, we need to test the following null hypothesis:
H_0: b = 0
H_1: b ≠ 0

The critical value of t for 28 – 5 = 23 degrees of freedom and a 5% significance level is 2.069 (from the t-table), while the t-ratio for $\hat{b}$ is 2100/340 = 6.1765 and is much greater than the critical value. (The rejection rule is such that the null is rejected when the absolute value of the t-statistic exceeds the critical value). This leads us to reject the null hypothesis which implies that $\hat{b}$ is significant or equivalently that the time trend enters the regression significantly.

Using p-values leads to a similar result.
The p-value for the time trend (0.0001) indicates strong statistical significance as it is much smaller than α = 0.05. Thus, the regression analysis provides very strong statistical evidence of an upward trend in shoe sales.

b) Looking at the t-tests (at α = 5% = 0.05 significance level, the critical value being 2.069) we have:

For D_1, the null and alternative hypotheses are:
H_0: c_1 = 0
H_1: c_1 ≠ 0

The t-statistic is: t = 3280/1510 = 2.17

Since t –statistic = 2.17 > 2.069 we reject the null hypothesis meaning that the dummy variable associated with the first quarter, D_1, is significant.

For D_2, the null and alterative hypotheses are:
H_0: $c_2 = 0$
H_1: $c_2 \neq 0$

The t-statistic is t = 2.82 > 2.069 meaning that we reject the null and that D_2 is significant.

For D_3, the null and alterative hypotheses are:
H_0: $c_3 = 0$
H_1: $c_3 \neq 0$

The t-statistic is = 4.44 > 2.069 meaning that D_3 is significant.
Testing the joint significance of all the variables included in the regression amounts to testing the null and alternative hypotheses that follow:

H_0: $a = b = c_1 = c_2 = c_3 = 0$
H_1: H_0 is not true

To do that, we can compute the F-statistic as:

$$F = \frac{R^2/(k-1)}{(1-R^2)/(n-k)}$$

With n = 28, k = 5 and $R^2 = 0.9651$. Doing the computations yields an F-statistic of 159.01 which far exceeds the critical value of 2.80 coming from the F-table with α= 5%. This implies that the variables are jointly significant.

Since all seasonal dummy variables are significantly different from zero, the data suggest that athletic shoe sales tend to exhibit a seasonal pattern. Individual and joint significance tests point to an important seasonal pattern in sales. Ceteris paribus, sales are expected to be 3,280 units higher in quarter 1 than in quarter 4, 6,250 units higher in quarter 2 than in quarter 4, and 7,010 units higher in quarter 3 than in the quarter 4.

c) For 2008Q3 and 2009Q2, t = 31 and 34, respectively.

$Q_{2008Q3} = 184{,}500 + 2{,}100 \times (31) + 3{,}280 \times (0) + 6{,}250 \times (0) + 7{,}010 \times (1) = 256{,}610$ units

$Q_{2009Q2} = 184{,}500 + 2{,}100 \times (34) + 3{,}280 \times (0) + 6{,}250 \times (1) + 7{,}010 \times (01) = 262{,}150$ units

d) You might be able to improve this forecast equation by adding some additional explanatory variables that affect sales such as the price of the good, income, or a measure of aggregate economic activity.

Problem 18

Year	Actual Demand	3-Year Moving Average	Exponential Smoothing (w = 0.3)
1995	800	NA	NA
1996	925	NA	800.00
1997	900	NA	837.50
1998	1,025	875	856.25
1999	1,150	950	906.87
2000	1,160	1025	979.81
2001	1,200	1112	1033.87
2002	1,150	1170	1083.71

a) The first forecast, for 1998, is obtained as F(1998)= (800+925+900)/3 = 875. The forecast for 1999, is F(1999) = (1025+900+925)/3 = 950. The remaining forecasts are obtained similarly. See table above for the other values of the forecasts.

b) The exponential smoothing forecasts are computed as:

$$F_{t+1} = wA_t + (1 - w)F_t$$

When w = 0.3 and the first forecast is set to the actual, the forecast for 1996 is simply 800 (the value of the actual demand from the previous year) and the recursion can be started there to get as an example a forecast for 1997 of:

$$F(1997) = 0.3 \times 925 + 0.7 \times 800 = 837.5$$

and so on.
See the table for the other values.

c) The criterion used to compare forecast accuracy is the root mean squared error (RMSE) given by:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(A_t - F_t)^2}{n}}$$

where A_t denotes the actual (realized) value of the series, F_t denotes a forecast coming from one of the competing methods (moving averages, exponential smoothing) and n denotes the number of forecasts. The forecasting method yielding the lowest RMSE would be the best. In other words, when comparing two forecasting methods, a lower RMSE implies a better forecasting method.

d) For the 3-year moving average forecasting method:

Year	Actual Demand	3-Year Moving Average	$(A - F)^2$
1995	800	NA	NA
1996	925	NA	NA
1997	900	NA	NA
1998	1,025	875	22,500
1999	1,150	950	40,000
2000	1,160	1025	18,225
2001	1,200	1112	7,744
2002	1,150	1170	400

RMSE = 133.318

For the exponential smoothing forecasting method:

Year	Actual Demand	Exponential Smoothing (w = 0.3)	(A – F)²
1995	800	NA	NA
1996	925	800.00	15,625
1997	900	837.50	3,906.25
1998	1,025	856.25	28,476.56
1999	1,150	906.87	59,112.20
2000	1,160	979.81	32,468.44
2001	1,200	1033.87	27,599.18
2002	1,150	1083.71	4,394.36

RMSE = 156.56

Since the RMSE from the 3-year moving average is lower than the RMSE from the exponential smoothing method, the 3-year moving average forecasting method does better.

Problem 19

$$Q = 7{,}000 - 550P + 210I + 425P_C$$

a)

$$Q = 7{,}000 - 550 \times 3 + 210 \times 15 + 425 \times 4 = 10{,}200$$

The expected quantity demanded for the coming year is 10,200,000 pounds of tea.

b)

$$Q = 7{,}000 - 550 \times 3 + 210 \times 13 + 425 \times 7 = 11{,}055$$

The expected quantity demanded for the coming year is 11,055,000 pounds of tea.

Problem 20

$$\hat{Y} = 8.25 + 0.125t - 2.75D_{1t} + 2.25D_{2t} + 3.50D_{3t}$$

Quarter 1, 2006: t =21: D_1=1, D_2= 0, D_3=0
Y2006Q1 =8.25 + .125(21) − 2.75(1) + 2.25(0) + 3.50(0) = $8.125 (million)

Quarter 2, 2006: t = 22: D_1=0, D_2=1, D_3=0
Y2006Q2 = 8.25 + .125(22) − 2.75(0) + 2.25(1) + 3.50(0) = $13.25 (million)

Quarter 3, 2006: t = 23: D_1=0, D_2=0, D_3=1
Y2006Q3 = 8.25 + .125(23) − 2.75(0) + 2.25(0) + 3.50(1) = $14.625 (million)

Quarter 4, 2006: t = 24: D_1=0, D_2=0, D_3=0
Y2006Q4 =8.25 + .125(24) −2.75(0) + 2.25(0) + 3.50(0) = $11.25 (million)